

Fusion in stars and on Earth

Tokamaks and the Hot CNO cycle

The individual components of the sessions can be adjusted for level and time depending on what is included, what information is given in advance, and the level of detail in response expected from the group work.

Curriculum links

- Fusion reactions and fusion energy
- The statistical nature of beta decay and half-life
- Stars, energy generation in stars and stellar explosions
- Catalytic processes in general

Prior knowledge

It is recommended that students have some understanding of these topics before the activities:

- Nuclear particles: neutrons, protons and alpha particles
- Nuclear isotopes (for example several isotopes of hydrogen and oxygen)
- Beta decay and half-life
- Alpha decay

A brief introduction or recap of the following areas of particle physics is advisable before carrying out the activities:

- The constituents of a deuteron, an alpha particle and carbon-12 [covering nuclear particles].
- The two stable isotopes of hydrogen, helium and carbon (as well as their natural abundances), covering nuclear isotopes and their different constituents.
- The beta decay and half-life of, for example, fluorine-18 [covering beta decay and half-life]. This is the fluorine isotope used for cancer diagnostics in PET scans and has a half-life of 110 minutes.

Activity 1: Introduction to the physics of fusion (30 min)

Video

Show the film Born to Engineer – Kim Cave-Ayland – which introduces fusion <https://www.stem.org.uk/rx6zw6>

The binding energy of deuterium (heavy hydrogen) is the energy you gain when fusing a neutron and a proton (the nucleus of a regular hydrogen atom). It is similarly the energy it takes to tear them apart again. This can be modelled using two tennis balls stuck together with a double-sided Velcro pad. It is useful to use a blue ball for the neutron and a red ball for the proton.

In fusion tokamaks here on Earth, such as the one shown in the video, we fuse two versions of heavy hydrogen: deuterium (one proton and one neutron) and tritium (one proton and three neutrons). This creates ${}^4\text{He}$ [read as helium-4], which contains two protons and two neutrons while the excess neutron is expelled. The neutron is subsequently reused to produce tritium from lithium.

Materials

Make several tennis ball deuterium nuclei (${}^2\text{H}$) and in groups let the students work on inventing methods for measuring how much energy it takes to tear the nuclei apart. Solutions they come up with might include: measuring the force you need to apply as you pull the pair apart (using a Newton meter), or knocking them apart by bouncing them off a wall and measuring how fast they come off the wall using a high-speed video camera.

After having discussed the solutions with individual groups, the teacher could ask one (or two) of the groups to present their idea to the class, otherwise the ideas can be discussed with input from all groups.

Activity 2: Calculations of fusion energy (35 min)

Introduction (5 min)

The energy you gain from fusing two atomic nuclei (or the energy it takes to tear them apart again) corresponds to a tiny change in their mass. By becoming slightly lighter, the atomic nuclei emit the rest of their mass as energy, which can be calculated through Einstein's formula: $E=mc^2$. The mass, and therefore excess energy, available compared to carbon-12 (mass is 12 atomic mass units, u) in the following atomic nuclei is given below. The energy unit TJ (tera joules) is 10¹² joules.

Calculations of fusion energy (20 min)

Nucleus with number of nucleons (A=1, 2, 4, 12) as superscript	Mass of nucleus in atomic mass units ($u = 1.661 \times 10^{-27}$ kg)	Excess energy compared to carbon in units of uc^2 ($uc^2 = 1.492 \times 10^{-10}$ J)	Excess energy per kg material (dividing the excess energy through by $A \times u$)
¹ H (hydrogen-1)	$m = 1.007825$ u	$E = 0.007825$ uc^2	703 TJ/kg
² H (hydrogen-2)	$m = 2.014102$ u	$E = 0.014102$ uc^2	[633 TJ/kg]
⁴ He (helium-4)	$m = 4.002603$ u	$E = 0.002603$ uc^2	58 TJ/kg
¹² C (carbon-12)	$m = 12.000000$ u	$E = 0.000000$ uc^2	[0 TJ/kg]

In groups of four:

- Calculate the excess energy per kg material for neutrons, ²H (also known as deuterium) and ¹²C, missing in the above table.
- Write down the reaction process [fusion material(s) \rightarrow fusion product] and calculate the energy you gain in fusion per kg of material for at least one of the following reactions. It is useful to decide in advance which group will present which of the four fusion reactions (a to c) to allow each group to focus only on one reaction.
 - Calculate fusion of two deuterium nuclei (also known as heavy hydrogen or hydrogen-2, ²H), creating helium (⁴He). This is relatively similar to the fusion process used in tokamaks, with the exception that no neutron is emitted. [575 TJ/kg]
 - Calculate fusion of four regular hydrogen nuclei to create helium (⁴He). This is the process that powers our sun (and other stars). The process also involves beta decay processes, but the energy produced can still be calculated from their excess energy as above. [645 TJ/kg]¹
 - Fusion of three helium (⁴He) nuclei to one carbon nucleus (¹²C). This process powers the majority of stars towards the end of their life. [58 TJ/kg]

The following can be included if additional time is added to the session, as homework, or for discussion with the results given by the teacher:

The total UK electricity generation in 2015 was 338TWh (population of 65.1 million).

- How long would one kg of material cover your electricity needs, using the fusion reaction you have focused on above? You may assume that you use an average share of the generated electricity in the UK.
- Using that 0.0312% of the mass of hydrogen is deuterium (hydrogen-2), calculate for the reaction in question (2) approximately how much sea water you need to extract deuterium from in order to supply you with one year's worth of electricity. How long could the seas supply the world with electricity using this method? (You will need to search online for estimates of the amount of seawater in the world.)

Group presentation (10 min)

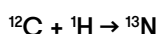
While the teacher takes notes on the board, groups present their results for either (1) or their focus reaction (2) and possibly (3). If more than one group has worked on the same reaction, different groups may contribute to (2) and (3) for each reaction.

¹ In our sun, this fusion process takes place through the so-called pp-chains (named after the proton-proton fusion reactions involved). The first of these involves the reaction: ${}^1\text{H} + {}^1\text{H} \rightarrow {}^2\text{H}$ (deuterium) + positron, where two hydrogen nuclei fuse while at the same time undergoing beta (+) decay. This is followed by proton absorption on deuterium: ${}^2\text{H} + {}^1\text{H} \rightarrow {}^3\text{He}$. When two such helium-3 isotopes have been produced, they can fuse in the reaction: ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + {}^1\text{H} + {}^1\text{H}$. Note that in the beta (+) decays here and in the following a neutrino is also emitted, in addition to the positron. This has been omitted here for simplicity, but could be introduced to the most advanced students.

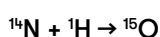
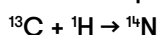
Activity 3: Modelling the hot CNO cycle (40 min)

Introduction (5 min)

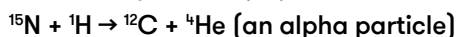
The CNO cycles in stars represent the predominant type of hydrogen fusion in the universe. These cycles particularly dominate hydrogen fusion for stars that are heavier than 1.3 times the mass of our sun. The process is a catalytic fusion process, facilitated particularly by isotopes of carbon, nitrogen and oxygen (CNO). These function as catalysts in the same way as chemical processes can be sped up by catalysts (for example in the exhaust of a car). The catalytic fusion process facilitates hydrogen fusion throughout the stars' lives, and speeds up the fusion process compared to the pp-chains present in our sun. A key example of catalytic hydrogen fusion is the first CNO cycle:



$^{13}\text{N} \rightarrow ^{13}\text{C} + \text{positron (e+)}$, with a beta decay half-life of approximately 10 minutes



$^{15}\text{O} \rightarrow ^{15}\text{N} + \text{positron (e+)}$, with a beta decay half-life of approximately 2 minutes



The cycle involves four absorptions of a hydrogen-1 nucleus (a proton) as well as two beta (+) decays. All of the six stages contribute to the emitted energy, a total of 645TJ/kg released energy, as calculated in the previous activity 2(b).

The Hot CNO cycle facilitates catalytic hydrogen fusion in very hot stars, in particular stellar explosions such as novae and X-ray bursts. This similarly converts four 1H nuclei to 4He. For the hottest versions of the cycle isotopes of fluorine and neon also play an important role. While all of the curriculum links listed above can be discussed based on the activity, the activity has been particularly designed to give a hands-on understanding of the statistical nature of nuclear decay and nuclear half-life.

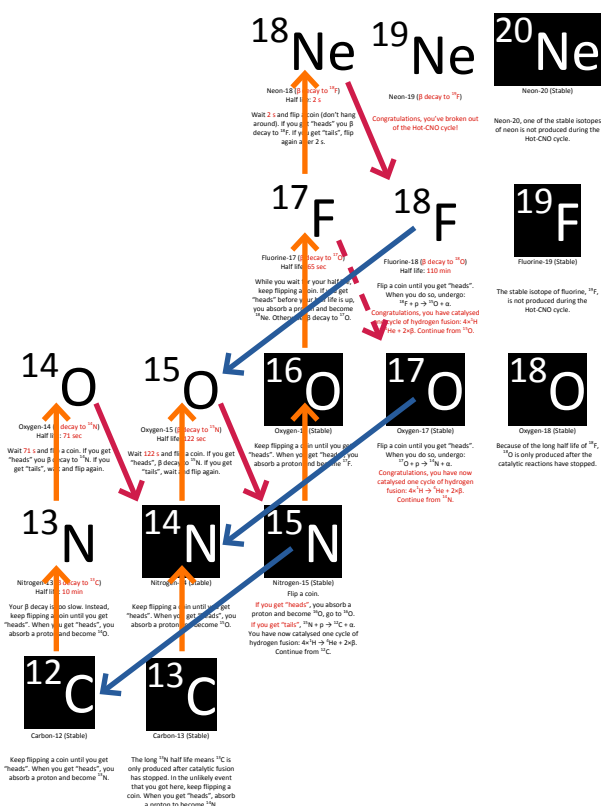
Modelling the hot CNO cycle (20 min)

Materials

- A3 printouts of Hot-CNO cycle isotope boards (preferably laminated). An open floor space of 4m by 4m, preferably with additional space for observers
- Blu-tack or tape for sticking the boards to the floor
- Stopwatch and a coin per student

Preparation

The boards are laid out on the floor in the order shown below, with approximately 20-30cm between the boards. 10 to 15 students can be engaged with the activity at any given time, so a typical school class should be divided into two groups. One group will observe the other for 5 to 10 minutes and the groups then swap.



Procedure

Each student takes the role of a catalysing nucleus. Initially, the students should each stand on one of the isotopes: ^{12}C , ^{13}N , ^{13}C , ^{14}N , ^{15}O , ^{15}N or ^{16}O . While they should each stand so they can touch their respective isotope with their foot, several are allowed to stand on each isotope at the same time.

Using a stopwatch and a coin students move around the board as they catalyse hydrogen fusion reactions and beta decays as instructed on the boards. The possible steps are indicated by arrows in the diagram above (the students will be reproducing this later based on their experience; see activity 4).

Stable isotopes are indicated by black squares with white text, unstable (beta-decaying) isotopes by white squares with black text. Since in some cases proton absorption is faster than beta decay, not all unstable isotopes will make it to their beta decay but will bypass it through proton absorption.

In the diagram, beta decays are indicated by red arrows and will require the student to wait for one half-life before flipping a coin to determine whether to decay or wait another half-life.

Proton (^1H) absorptions [orange arrows] and proton-absorption followed by alpha (^4He) emission [blue arrows] are fast and will happen as soon as the student manages to flip a coin to get heads (if necessary repeating the flipping of the coin as fast as practical).

For example, if you stand on ^{15}O (that is you are at the moment a ^{15}O nucleus): you must now wait during its 122-second half-life and then flip a coin. If the coin shows heads up, you will beta decay to ^{14}N . If the coin shows tails, you have to wait for another half-life and try again (and again, and again... some students will get stuck for many half lives, which is part of the learning experience).

Other nuclei – such as, for example, ^{12}C – will immediately fuse with a proton (the ^1H nucleus). This happens as soon as the student gets heads from repeated flipping of their coin and the student will in this case become ^{13}N , as indicated by the arrow from ^{12}C to ^{13}N . For each time a full cycle has been completed (for example through $^{17}\text{O} + ^1\text{H} \rightarrow ^{14}\text{N} + ^4\text{He}$) one helium nucleus (alpha particle) has been produced.

Activity 4: Group work and class discussion of the Hot CNO cycle (15 min)

Materials

- A3 printouts of Hot CNO overview for each group
- Pens of three different colours

Each group of four draws arrows on their A3 overview to indicate all of the different steps of the Hot CNO cycle. It is useful to use different colours for beta (β^-) decay, proton (^1H) absorption and proton (^1H) absorption with immediate alpha (^4He) emission.

Suggested discussion points (in groups and as a class)

- How many of you completed a full Hot CNO cycle (ie produced an alpha particle)?
- How many of you got stuck on either ^{14}O or ^{15}O for a long time (several half-lives)? [Often one or two in the class will never have got past the two oxygen isotopes.]
- How much energy is generated from the conversion of four hydrogen atoms to one helium atom? [See also activity 2, Q4.]
- Does this change when you add CNO as catalysts? [The energy budget is unaffected by the presence of a catalyst, as in, for example, $4 \times ^1\text{H} + ^{15}\text{O} \rightarrow ^4\text{He} + ^{15}\text{O}$ representing one of the cycles has the mass of ^{15}O retained both before and after the reaction.]²

Additional material for teacher-led discussion

How can it be that you didn't just absorb another proton (^1H) while you were stuck at ^{14}O and ^{15}O ? You can also ask them as a step along the way: what would the resulting nuclei be in such processes?

Answer: the resulting nuclei ^{15}F and ^{16}F produced when absorbing a proton onto either of the two oxygen isotopes decay immediately by proton emission, rather than beta decay, and you therefore get back to oxygen immediately, as if you never left. This is because ^{15}F and ^{16}F are 'unbound' ie the last proton is not stuck to the nucleus. The nucleus therefore gains energy by emitting the proton. The process is much faster than the beta decay (typically of the order 10-20 seconds, rather than the beta decay half-life of some seconds). The proton emission process is conceptually the same as the alpha decay we observe for some of the heavy elements, but is much faster than typical alpha decay half-lives because the charge of the involved nuclei is much smaller in the case of proton decay of ^{15}F and ^{16}F .

² Note that in the above, the reactions converting $4 \times ^1\text{H}$ to ^4He have been simplified to exclude neutrinos. If neutrinos are included (two neutrinos are emitted for each full cycle), it should be noted that they will carry away some of the energy. The energy budget is unaffected by this, and the effect is present in all types of stellar hydrogen fusion. However, the amount of energy carried away by the neutrinos does depend on whether the fusion

Acknowledgements

Idea and design: Christian A. Diget [University of York] and Adam Tuff [Kromek Group plc]. Please email comments and ideas for revised material to christian.diget@york.ac.uk. Particularly if you implement modifications to the material, have feedback on its use and incorporation into GCSE and/or A level, or if you use the material in conjunction with other teaching material. This will assist the further development of the material.